

4. A particle moves along the  $x$ -axis with position at time  $t$  given by  $x(t) = e^{-t} \sin t$  for  $0 \leq t \leq 2\pi$ .
- (a) Find the time  $t$  at which the particle is farthest to the left. Justify your answer.
- (b) Find the value of the constant  $A$  for which  $x(t)$  satisfies the equation  $Ax''(t) + x'(t) + x(t) = 0$  for  $0 < t < 2\pi$ .

$t$ (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5.

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function  $r$  of time  $t$ , where  $t$  is measured in minutes. For  $0 < t < 12$ , the graph of  $r$  is concave down. The table above gives selected values of the rate of change,  $r'(t)$ , of the radius of the balloon over the time interval  $0 \leq t \leq 12$ . The radius of the balloon is 30 feet when  $t = 5$ .
- (Note: The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .)
- (a) Estimate the radius of the balloon when  $t = 5.4$  using the tangent line approximation at  $t = 5$ . Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when  $t = 5$ . Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_0^{12} r'(t) dt$ ? Give a reason for your answer.